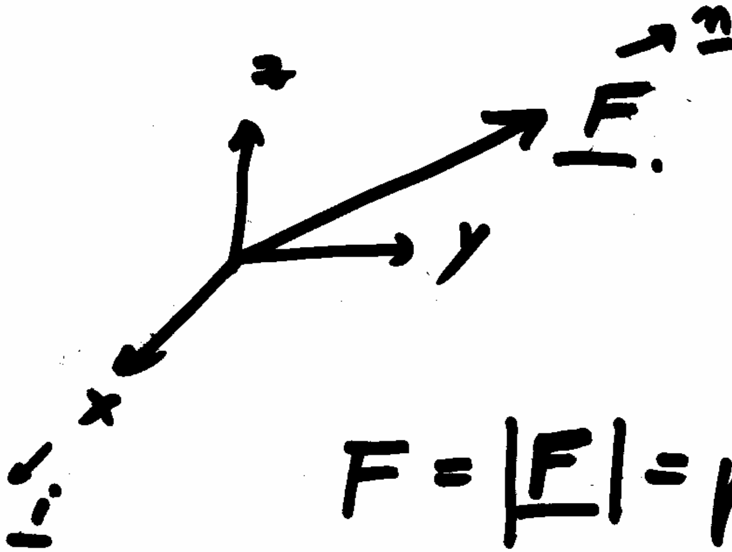
# STATICS:

# A REVIEW

# FORCE

$$\boxed{\underline{F} = F_x \underline{i} + F_y \underline{j} + F_z \underline{k} = F \underline{n}}$$




$$\boxed{\underline{F} = F \underline{n}}$$

$$F = |\underline{F}| = \sqrt{F_x^2 + F_y^2 + F_z^2}$$

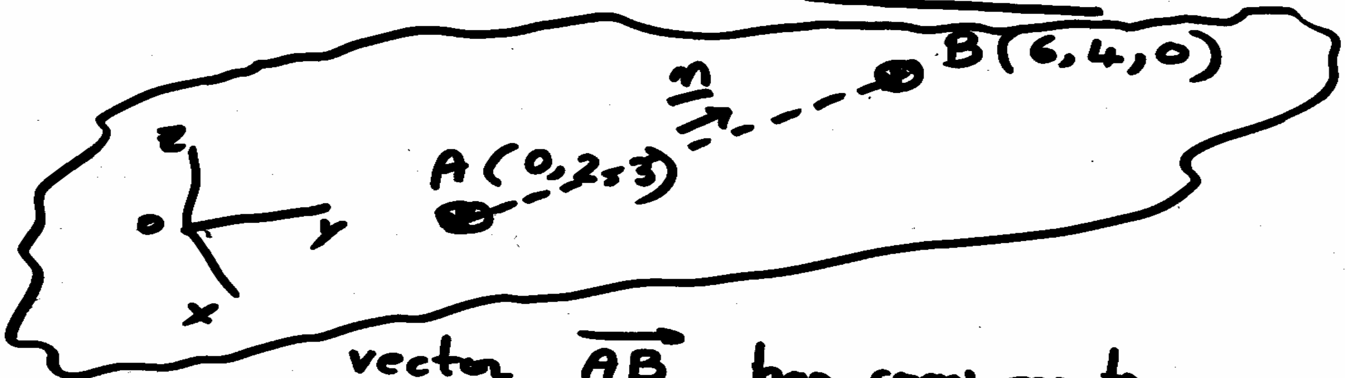
magnitude $|\underline{n}| = \sqrt{n_x^2 + n_y^2 + n_z^2} = 1$ = unity

$$\boxed{\underline{n} = n_x \underline{i} + n_y \underline{j} + n_z \underline{k}}$$

$$|\underline{i}| = 1, \quad |\underline{j}| = 1, \quad |\underline{k}| = 1$$

$$\underline{F} = \vec{F} = 700\,\underline{n}$$

A Force is a vector — Magnitude, Direction $\Rightarrow \underline{F}$




vector $\vec{AB}$ has components

$\Delta X = 6 - 0 = 6$

$\Delta Y = 4 - 2 = 2$

$\Delta Z = 0 - 3 = -3$

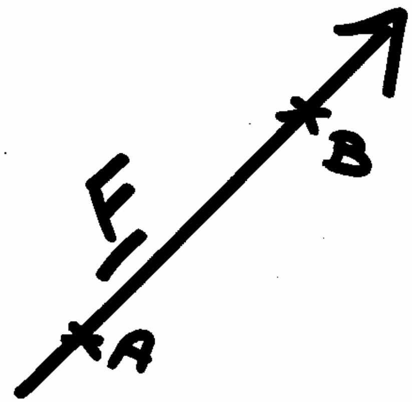
$$\vec{AB} = 6\underline{i} + 2\underline{j} - 3\underline{k}$$

magnitude of $|\vec{AB}| = \sqrt{(6)^2 + (2)^2 + (-3)^2}$

$$\therefore |\vec{AB}| = 7$$

unit vector along AB is

$$\vec{n} = \frac{\vec{AB}}{|\vec{AB}|} = \frac{1}{7}\left[6\underline{i} + 2\underline{j} - 3\underline{k}\right] = \frac{6}{7}i + \frac{2}{7}j - \frac{3}{7}k$$




$$\vec{F} = F\,\vec{n}$$

$$\underline{F} = F\,\underline{n}$$

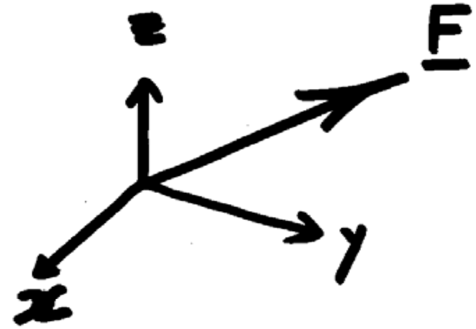
magnitude of force $= F = 700$ lbs

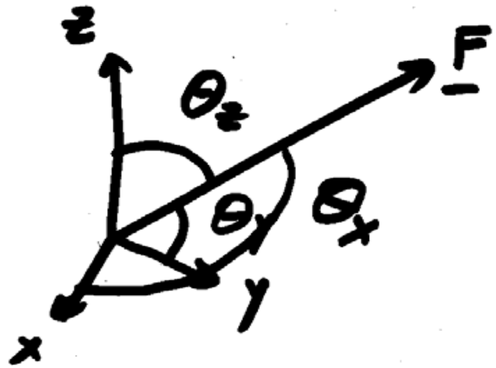
direction $\underline{n} = \frac{6}{7}\underline{i} + \frac{2}{7}\underline{j} - \frac{3}{7}\underline{k}$

$$\underline{F} = 700\left[\frac{6\underline{i} + 2\underline{j} - 3\underline{k}}{7}\right] = 600\underline{i} + 200\underline{j} - 300\underline{k}$$

$$= F_x\underline{i} + F_y\underline{j} + F_z\underline{k}$$

# Direction Cosines




$$\underline{F} = F_x \underline{i} + F_y \underline{j} + F_z \underline{k}$$

$$\cos\theta_x = \frac{F_x}{F} = \frac{600}{700} = \frac{6}{7} = n_x$$

$$\cos\theta_y = \frac{F_y}{F} = \frac{200}{700} = \frac{2}{7} = n_y$$

$$\cos\theta_z = \frac{F_z}{F} = \frac{-300}{700} = -\frac{3}{7} = n_z$$

$$\cos^2\theta_x + \cos^2\theta_y + \cos^2\theta_z = 1$$

$$n_x^2 + n_y^2 + n_z^2 = 1$$

$\underline{M} = M_x \underline{i} + M_y \underline{j} + M_z \underline{k}$

Moment is also a vector

$M_x = \underline{M} \cdot \underline{i}$

$= |\underline{M}| \cdot |\underline{i}| \cos(\underline{M}, x)$

$\underline{i} \cdot \underline{j} = (1)(1) \cos 90$

$= 0$

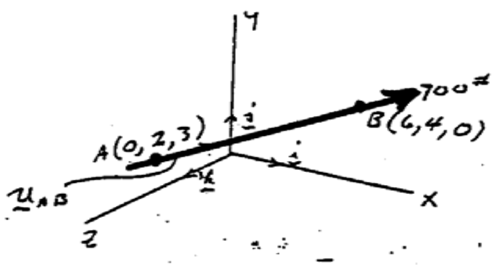
$\underline{i} \cdot \underline{i} = 1$

$\underline{F} \cdot \underline{F} = F^2$

# Example Problems with Solutions for Statics

## A. Forces and Moments

1. Given: A force of 700# magnitude passes through points A and B as shown.




$\underline{n} = \underline{u}_{AB}$

$\underline{F} = 700\underline{n}$

Find: a) Express this force as a vector using unit vectors.

b) What are the x-, y-, and z-components of the force?

c) Give the direction cosines of a line segment drawn from A to B.

**Solution:**

a) $\underset{\sim}{F} = 700\underset{\sim}{u}_{AB}$ where $\underset{\sim}{u}_{AB}$ is a unit vector along line AB.

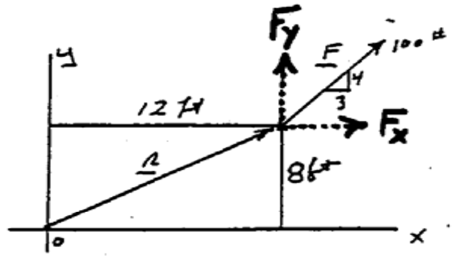
$$= 700\underbrace{\left[\frac{(6-0)\underset{\sim}{i}+(4-2)\underset{\sim}{j}+(0-3)\underset{\sim}{k}}{\sqrt{(6-0)^2+(4-2)^2+(0-3)^2}}\right]}_{\underset{\sim}{u}_{AB}} = 700\left[\frac{6\underset{\sim}{i}+2\underset{\sim}{j}-3\underset{\sim}{k}}{7}\right]$$

or $\boxed{\underset{\sim}{F} = 600\underset{\sim}{i} + 200\underset{\sim}{j} - 300\underset{\sim}{k}}$

b) $F_x = 600\#$, $F_y = 200\#$, $F_z = -300\#$

c) $\cos\theta_x = \frac{6}{7}$. $\cos\theta_y = \frac{2}{7}$, $\cos\theta_z = -\frac{3}{7}$

note: $F_x = |\underset{\sim}{F}|\cos\theta_x = 700 \times \frac{6}{7} = 600$, etc.

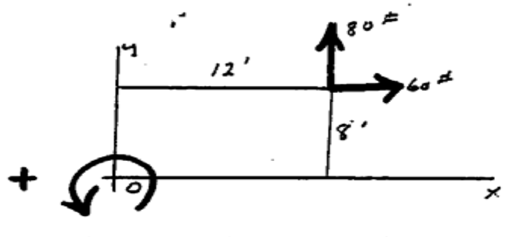



2. Given: A force of 100# acts at point (12,8) as shown.

Find: The moment of the force with respect to the z-axis.

Solution:

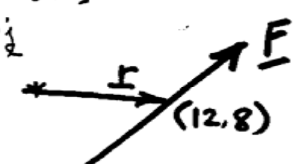
Resolve the force into components $F_x = \frac{3}{5} * 100 = 60\#$ and $F_y = \frac{4}{5} * 100 = 80\#$




$$\sum M_z = 80 \times 12 - 60 \times 8$$
$$= 960 - 480 = 480 \text{ ft}\#$$
$$M_z = 480 \text{ ft}\#$$
or $\boxed{\underset{\sim}{M_z} = 480\, \underset{\sim}{k} \text{ ft}\#}$

Alternate solution:

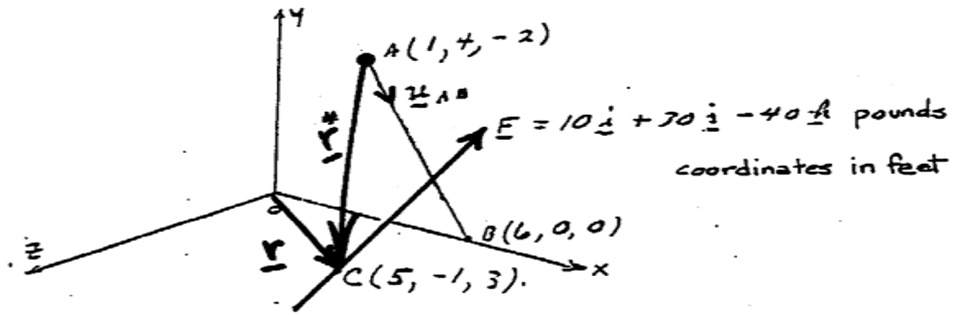
Vectorize the force as $\underline{F} = 100\left[\frac{3}{5}\underset{\sim}{i} + \frac{4}{5}\underset{\sim}{j}\right]$
$= 60\underset{\sim}{i} + 80\underset{\sim}{j}$




$$\underset{\sim}{M_o} = \underset{\sim}{r} \times \underset{\sim}{F} = (12\underset{\sim}{i} + 8\underset{\sim}{j}) \times (60\underset{\sim}{i} + 80\underset{\sim}{j})$$
$$= 960\underset{\sim}{k} - 480\underset{\sim}{k} = 480\underset{\sim}{k}$$

$\boxed{\underset{\sim}{M_o} = 480\,\underset{\sim}{k}}$ (same as above)

$$\underline{M_o} = \underline{r} \times \underline{F}$$
$$M_o = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$




3. **Given:** The force $\mathbf{F} = 10\mathbf{i} + 30\mathbf{j} - 40\mathbf{k}$ passes through point C(5,-1,3).

**Find:**
a) Moment of $\mathbf{F}$ with respect to the origin.
b) Moment of $\mathbf{F}$ with respect to point A.
c) Moment of $\mathbf{F}$ with respect to line AB.
d) As seen from looking from B to A, is the moment of part c) clockwise or counterclockwise?

$$\overrightarrow{AB} = 5\mathbf{i} - 4\mathbf{j} + 2\mathbf{k}\,,$$

<u>Solution:</u>

$$|\overrightarrow{AB}| = \sqrt{25+16+4} = \sqrt{45}$$

a) $\mathbf{M}_o = \mathbf{r}_{oc} \times \mathbf{F} = (5\mathbf{i} - \mathbf{j} + 3\mathbf{k}) \times (10\mathbf{i} + 30\mathbf{j} - 40\mathbf{k})$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 5 & -1 & 3 \\ 10 & 30 & 40 \end{vmatrix} = \mathbf{i}(40-90) - \mathbf{j}(-200-30) + \mathbf{k}(150+10)$$

$$= \boxed{-50\mathbf{i} + 230\mathbf{j} + 160\mathbf{k} = \mathbf{M}_o}$$

b) $\mathbf{M}_A = \mathbf{r}_{AC} \times \mathbf{F} = [(5-1)\mathbf{i} + (-1-4)\mathbf{j} + (3+2)\mathbf{k}] \times (10\mathbf{i} + 30\mathbf{j} - 40\mathbf{k})$

$$= (4\mathbf{i} - 5\mathbf{j} + 5\mathbf{k}) \times (10\mathbf{i} + 30\mathbf{j} - 40\mathbf{k})$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 4 & -5 & 5 \\ 10 & 30 & -40 \end{vmatrix} = \mathbf{i}(200-150) - \mathbf{j}(-160-50) + \mathbf{k}(120+50)$$

or $\boxed{\mathbf{M}_A = 50\mathbf{i} + 210\mathbf{j} + 170\mathbf{k}}$

c) $\mathbf{M}_{AB} = (\mathbf{r}_{AC} \times \mathbf{F}) \cdot \mathbf{u}_{AB} = (50\mathbf{i} + 210\mathbf{j} + 170\mathbf{k}) \cdot \left(\frac{5}{\sqrt{45}}\mathbf{i} - \frac{4}{\sqrt{45}}\mathbf{j} + \frac{2}{\sqrt{45}}\mathbf{k}\right)$

$$\mathbf{M}_{AB} = \mathbf{M}_A \cdot \mathbf{u}_{AB}$$

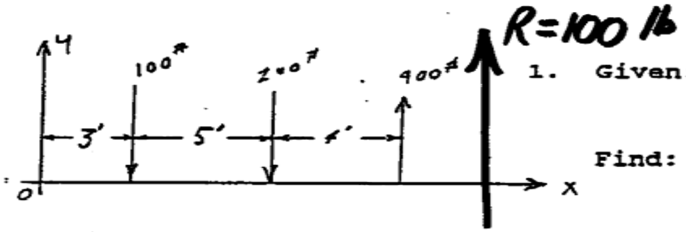
c) (cont.)

$$\underset{\sim}{M}_{AB} = \frac{250}{\sqrt{45}} - \frac{840}{\sqrt{45}} + \frac{340}{\sqrt{45}} = -\frac{250}{\sqrt{45}} = -\frac{250}{6.7} = -37.4$$

$$\boxed{\underset{\sim}{M}_{AB} = -37.4 \text{ \#ft}}$$

d) Clockwise in opposite direction to $\underset{\sim}{\mu}_{AB}$ by right hand rule.

B. **Resultants of Force Systems**



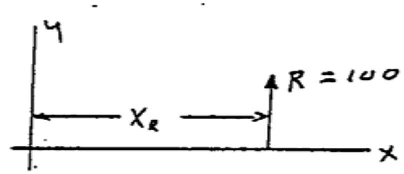

1. Given: The three coplanar parallel forces as shown.

Find: The resultant of this force system.

$$\begin{cases} R_x = \Sigma F_x \\ R_y = \Sigma F_y \\ R_z = \Sigma F_z \end{cases}$$

**Solution:**

$$\underset{\sim}{R} = 100\downarrow + 200\downarrow + 400\uparrow = 100\uparrow \quad (\text{or } \underset{\sim}{R} = 100\underset{\sim}{j})$$




In given system,

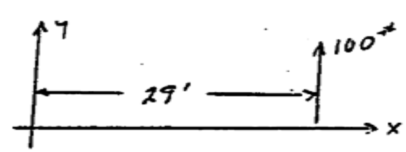
$$\curvearrowleft + \Sigma M_z = -100 \times 3 - 200 \times 8 + 400 \times 12$$
$$= -300 - 1600 + 4800 = +2900$$

For Resultant,

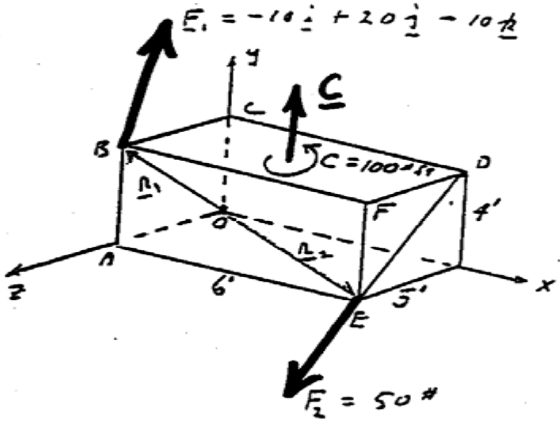
$$\curvearrowleft + \Sigma M_z = 100\, X_R$$

$$\therefore 100\, X_R = 2900$$

or $X_R = 29$ ft




ans.




2. Given: The force system shown, consisting of the two forces and one couple. $F_1$, passes through point B; $F_2$ has magnitude of 50# and passes through points D and E; the couple lies in plane BCDF and is counterclockwise as seen from above.

Find: The resultant of the given system expressed as a force at the origin plus a couple.

**Solution:**

$$R = (\Sigma F_x)i + (\Sigma F_y)j + (\Sigma F_z)k$$

$$= (F_{1x}+F_{2x})i + (F_{1y}+F_{2y})j + (F_{1z}+F_{2z})k$$

$$= (-10+0)i + \left[20 - \tfrac{4}{5}\times 50\right]j + \left[-10 + \tfrac{3}{5}\times 50\right]k$$

or $\boxed{R = -10i - 20j + 20k}$ ← force at origin

$$C_R = \Sigma R_i \times F_i + \Sigma C_i$$

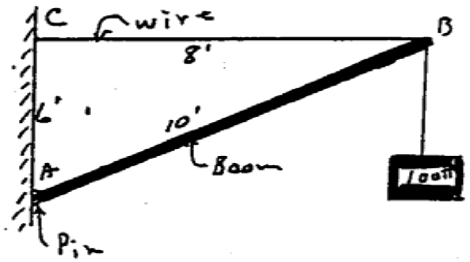
$$= R_1 \times F_1 + R_2 \times F_2 + C$$

$$= \left[(4j+3k)\times(-10i+20j-10k)\right] + \left[(6i+3k)\times(-40j+30k)\right] + \left[100\,j\right]$$

$$= \begin{vmatrix} i & j & k \\ 0 & 4 & 3 \\ -10 & 20 & -10 \end{vmatrix} + \begin{vmatrix} i & j & k \\ 6 & 0 & 3 \\ 0 & -40 & 30 \end{vmatrix} + 100j = \boxed{\begin{matrix} 20i - 110j - 200k \\ = C_R \end{matrix}}$$

couple (free vector)

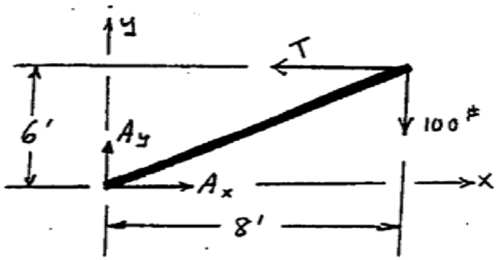
## C. Equilibrium




1. **Given:** A boom and wire assembly supports a 100 pound weight.

   **Find:** a) Tension in the wire.
   b) Pin reaction at A.

**Solution:** Free body diagram is the boom.




a) $\overset{+}{\curvearrowleft}\sum M_A = 0$

$$6T - 8 \times 100 = 0$$

$$\boxed{T = \frac{800}{6} = \frac{400}{3}}$$

b) $\overset{+}{\rightarrow}\sum F_x = 0$

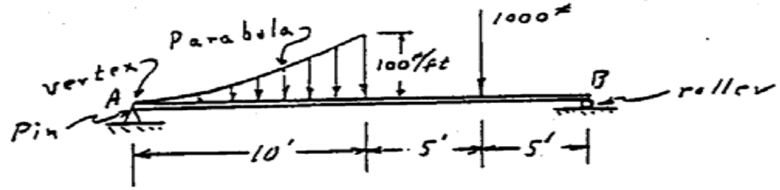
$$A_x - T = 0$$

$$\boxed{A_x = T = \frac{100}{3} \rightarrow}$$

$+\uparrow \sum F_y = 0$

$$A_y - 100 = 0$$

$$\boxed{A_y = 100 \uparrow}$$

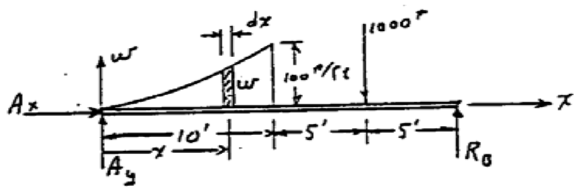



2. **Given:** A simply supported beam is loaded as shown with a parabolically distributed load on the left half and with a 1000# load 5 ft from the right end. The maximum intensity of the distributed load is 100 pounds per foot.

**Find:** The reactions at A and B.

<u>**Solution:**</u>

Free body diagram is the beam.




Equation of intensity-of-loading curve:

$w(x) = Kx^2$ ← parabola

@ $x = 10$, $w(10) = 100$

$\therefore 100 = K(10)^2$ or $K = 1$

Thus $w(x) = x^2$

$\overset{+}{\rightarrow}\Sigma F_x = 0$

$\boxed{A_x = 0}$

$dF = w(x)dx$

$dM_A = x\,w(x)dx = x\,w\,dx = x^3 dx$

$\overset{+}{\circlearrowleft}\Sigma M_A = 0$

$$20R_B - 1000\times 15 - \int_0^{10} x[w(x)dx] = 0$$

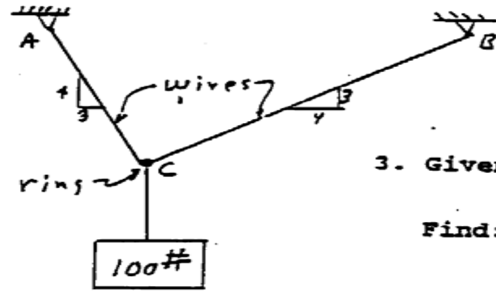
$$R_B = \frac{15000 + \int_0^{10} x(x^2)dx}{20} = \frac{15000 + \frac{x^4}{4}\Big]_0^{10}}{20} = \frac{15000 + 2500}{20} = \boxed{875\# = R_B}$$

$\overset{+}{\uparrow}\Sigma F_V = 0$

$$A_y + R_B - 1000 - \int_0^{10} w(x)dx = 0$$

$$A_y = 1000 + \int_0^{10} x^2 dx - R_B = 1000 + \frac{x^3}{3}\Big]_0^{10} - 875 = 125 + \frac{1000}{3} = \frac{1375}{3}$$

or $\boxed{A_y = \frac{1375}{3} = 458\#}$

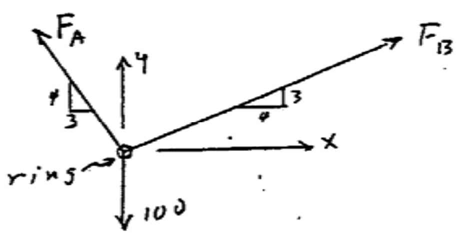



3\. **Given:** The two wires AC and BC support the 100 pound weight as shown.

**Find:** The tensile force in each wire.

**Solution:**

Free body is ring at C.




$$\overset{+}{\rightarrow}\Sigma F_x = 0$$

$$-\frac{3}{5}F_A + \frac{4}{5}F_B = 0 \qquad (1)$$

$$+\!\uparrow\Sigma F_y = 0$$

$$\frac{4}{5}F_A + \frac{3}{5}F_B - 100 = 0 \qquad (2)$$

Solve Equas. (1) + (2) simultaneously for $F_A$ + $F_B$:

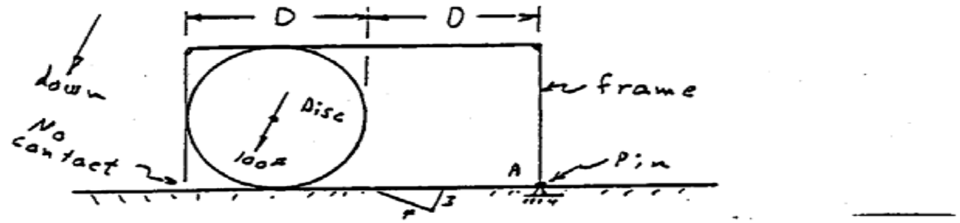
From (1) $\quad F_A = \frac{4}{3}F_B$

From (2) $\quad 4\left(\frac{4}{3}F_B\right) + 3F_B = 500$

$$\left(\frac{16}{3} + 3\right)F_B = 500$$

$$\frac{25}{3}F_B = 500 \quad \text{or} \quad \boxed{\begin{array}{l} F_B = \frac{3}{25} \times 500 = 60\# \\ F_A = \frac{4}{3} \times F_B = \frac{4}{3} \times 60 \\ \qquad = 80\# \end{array}}$$

+

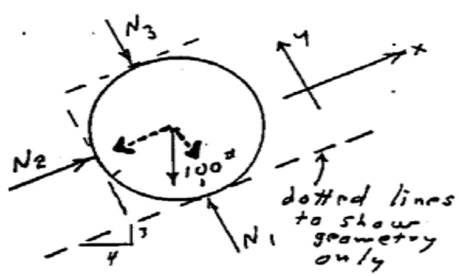



4. Given: A 100# disc is maintained in equilibrium on a 3-on-4 slope as shown. The frame is weightless and rigid. All surfaces are smooth.

Find: All unknown forces acting on the disc.

<u>Solution:</u>

Free body is the disc.




$\overset{+}{\nearrow}\ \Sigma F_x = 0$

$N_2 - \frac{3}{5}\times 100 = 0$

$\boxed{N_2 = 60\#}$ (1)

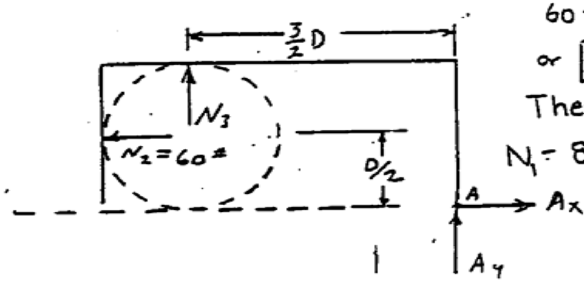
$\overset{+}{\nwarrow}\ \Sigma F_y = 0$

$N_1 - N_3 - 100\times\frac{4}{5} = 0$

or $N_1 - N_3 = 80\#$ (2)

No more <u>independent</u> equilibrium equations can be written for this concurrent coplanar system of forces. Therefore, we must use another free body.

The free body is the frame.




$\circlearrowleft + \Sigma M_A = 0$

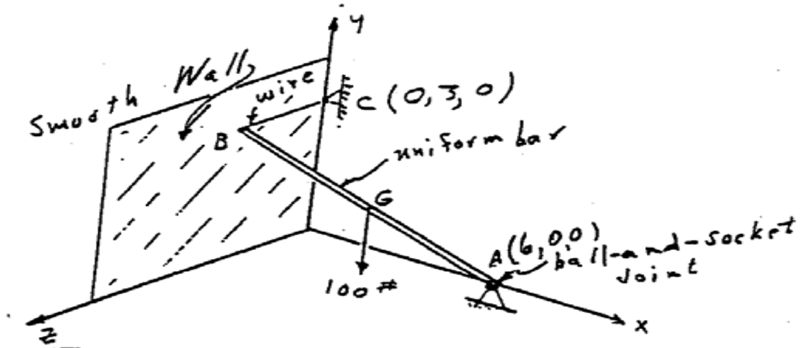
$60\times\frac{D}{2} - N_3\times\frac{3}{2}D = 0$

or $\boxed{N_3 = 20\#}$ (3)

Then from Eq. (2), we have

$N_1 = 80 + N_3 = 80 + 20 = 100\#$

$\boxed{N_1 = 100\#}$

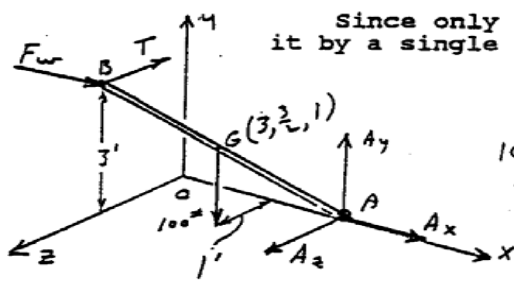



5. Given: A 100# uniform bar AB is supported by a ball-and-socket joint at point A(6,0,0) and by a smooth wall at point B(0,3,2). A wire BC prevents motion.

Find: The tension in the wire BC.

**Solution:**

Free body is the bar.




Since only one unknown is desired, we can find it by a single equilibrium equation.

$\Sigma M_x = 0$ (+ by right hand rule)

$100 \times 1 - 3T = 0$

or $\boxed{T = \frac{100}{3} = 33.3\#}$

Note: This problem could have been solved using vectors, but with more effort.

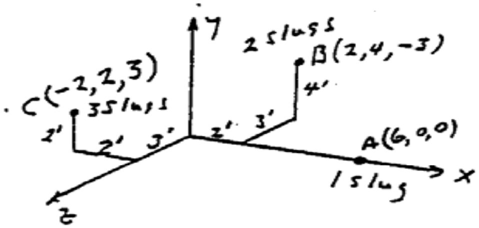
$$M_x = [\Sigma \underset{\sim}{r} \times \underset{\sim}{F}] \cdot \underset{\sim}{i} = 0$$

$$\text{or } \{[(0-6)\underset{\sim}{i} + (3-0)\underset{\sim}{j} + (2-0)\underset{\sim}{k}] \times (-T\underset{\sim}{k}) + [(3-6)\underset{\sim}{i} + (\tfrac{3}{2}-0)\underset{\sim}{j} + (1-0)\underset{\sim}{k}] \times (-100\underset{\sim}{j})\} \cdot \underset{\sim}{i} = 0$$

$$\text{or } [-6T\underset{\sim}{j} - 3T\underset{\sim}{i} + 600\underset{\sim}{k} + 100\underset{\sim}{i}] \cdot \underset{\sim}{i} = -3T + 100 = 0$$

$$\text{or } \boxed{T = \frac{100}{3} = 33.3}$$

## D. Centroids and Centers of Gravity




1. Given: Three discrete particles are located as shown. Masses are A = 1 slug, B = 2 slugs, and C = 3 slugs.

   Find: Location of center of mass of the system of particles.

**Solution:**

$$x_{cm} = \frac{\Sigma m_i x_i}{\Sigma m_i} = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3}{m_1 + m_2 + m_3}$$

$$= \frac{1\times 6 + 2\times 2 + 3\times(-2)}{1+2+3} = \frac{6+4-6}{6} = \frac{2}{3}'$$

$$y_{cm} = \frac{\Sigma m_i y_i}{\Sigma m_i} = \frac{m_1 y_1 + m_2 y_2 + m_3 y_3}{m_1 + m_2 + m_3}$$

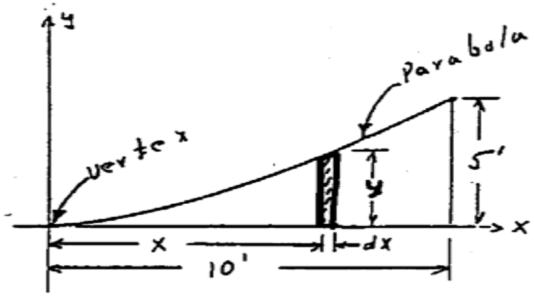
$$= \frac{1\times 0 + 2\times 4 + 3\times 2}{1+2+3} = \frac{8+6}{6} = \frac{14}{6} = \frac{7}{3}'$$

$$z_{cm} = \frac{\Sigma m_i z_i}{\Sigma m_i} = \frac{m_1 z_1 + m_2 z_2 + m_3 z_3}{m_1 + m_2 + m_3}$$

$$= \frac{1\times 0 + 2\times(-3) + 3\times 3}{1+2+3} = \frac{-6+9}{6} = \frac{3}{6} = \frac{1}{2}'$$

Center of mass is located @ $\left[x = \frac{2}{3}', y = \frac{7}{3}', z = \frac{1}{2}'\right]$

or $\boxed{\underset{\sim}{R}_{cm} = \frac{2}{3}\underset{\sim}{i} + \frac{7}{3}\underset{\sim}{j} + \frac{1}{2}\underset{\sim}{k}}$




$y = kx^2$

$y = 5'$ at $x = 10'$

$5 = k(10)^2$

$y = \frac{x^2}{20}$

$dA = y\,dx = \frac{x^2}{20}dx$

2. Given: A second degree parabola with vertex at origin passes through point (10,5).

Find: The location of the centroid of the area under the curve by direct integration.

<u>Solution:</u> $\bar{x} = \frac{\int x_c dA}{\int dA}$, $\bar{y} = \frac{\int y_c dA}{\int dA}$ [definition]

where $x_c$ = x-coordinate of the element (element considered as a rectangle) centroid.

$y_c$ = y-coordinate of the element centroid

First we must find the equation of the curve:

$y = Kx^2$ & @ $x = 10$, $y = 5 \Rightarrow 5 = K(10)^2$

or $K = \frac{5}{100} = \frac{1}{20}$

$\therefore y = \frac{x^2}{20}$

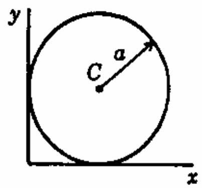
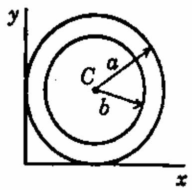
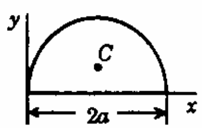
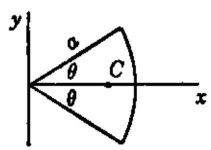
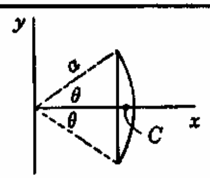
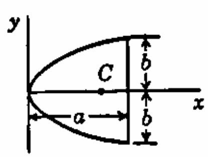
$$\therefore \bar{x} = \frac{\int x dA}{\int dA} = \frac{\int_0^{10} xy\,dx}{\int_0^{10} y\,dx} = \frac{\int_0^{10} x\left(\frac{x^2}{20}\right)dx}{\int_0^{10}\frac{x^2}{20}dx} = \frac{\int_0^{10} x^3 dx}{\int_0^{10} x^2 dx} = \frac{\left.\frac{x^4}{4}\right]_0^{10}}{\left.\frac{x^3}{3}\right]_0^{10}}$$

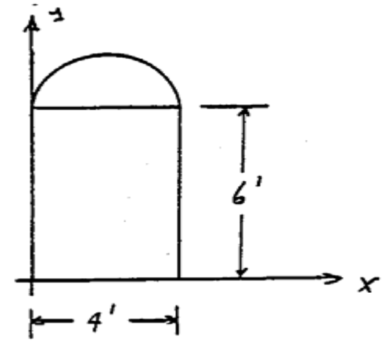
$$= \frac{3}{4}\times\frac{10^4}{10^3} = \frac{3}{4}\times 10 = 7.5' = \bar{x}$$

$$\bar{y} = \frac{\int y_c dA}{\int dA} = \frac{\int_0^{10}\left(\frac{y}{2}\right)y\,dx}{\int_0^{10}\frac{x^2}{20}dx} = 10\frac{\int_0^{10} y^2 dx}{\int_0^{10} x^2 dx} = \frac{10\int_0^{10}\frac{x^4}{400}}{\left.\frac{x^3}{3}\right]_0^{10}} = \frac{30}{400}\times\frac{\left[\frac{x^5}{5}\right]_0^{10}}{10^3}$$

$$= \frac{30\times 10^5}{400\times 5\times 10^3} = \frac{30\times 100}{2000} = \frac{3}{2}\text{ ft}$$

$$\boxed{\begin{array}{l}\bar{x} = 7.5\text{ ft}\\ \bar{y} = 1.5\text{ ft}\end{array}}$$

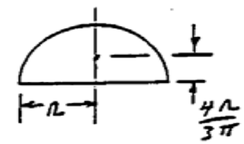
| Figure | Area & Centroid | Area Moment of Inertia | (Radius of Gyration)$^2$ | Product of Inertia |
|---|---|---|---|---|
|  | $A = \pi a^2$<br>$x_c = a$<br>$y_c = a$ | $I_{x_c} = I_{y_c} = \pi a^4/4$<br>$I_x = I_y = 5\pi a^4/4$<br>$J = \pi a^4/2$ | $r_{x_c}^2 = r_{y_c}^2 = a^2/4$<br>$r_x^2 = r_y^2 = 5a^2/4$<br>$r_p^2 = a^2/2$ | $I_{x_c y_c} = 0$<br>$I_{xy} = Aa^2$ |
|  | $A = \pi(a^2 - b^2)$<br>$x_c = a$<br>$y_c = a$ | $I_{x_c} = I_{y_c} = \pi(a^4 - b^4)/4$<br>$I_x = I_y = \dfrac{5\pi a^4}{4} - \pi a^2 b^2 - \dfrac{\pi b^4}{4}$<br>$J = \pi(a^4 - b^4)/2$ | $r_{x_c}^2 = r_{y_c}^2 = (a^2 + b^2)/4$<br>$r_x^2 = r_y^2 = (5a^2 + b^2)/4$<br>$r_p^2 = (a^2 + b^2)/2$ | $I_{x_c y_c} = 0$<br>$I_{xy} = Aa^2$<br>$= \pi a^2(a^2 - b^2)$ |
|  | $A = \pi a^2/2$<br>$x_c = a$<br>$y_c = 4a/(3\pi)$ | $I_{x_c} = \dfrac{a^4(9\pi^2 - 64)}{72\pi}$<br>$I_{y_c} = \pi a^4/8$<br>$I_x = \pi a^4/8$<br>$I_y = 5\pi a^4/8$ | $r_{x_c}^2 = \dfrac{a^2(9\pi^2 - 64)}{36\pi^2}$<br>$r_{y_c}^2 = a^2/4$<br>$r_x^2 = a^2/4$<br>$r_y^2 = 5a^2/4$ | $I_{x_c y_c} = 0$<br>$I_{xy} = 2a^2/3$ |
| <br>Circular Sector | $A = a^2\theta$<br>$x_c = \dfrac{2a}{3}\,\dfrac{\sin\theta}{\theta}$<br>$y_c = 0$ | $I_x = a^4(\theta - \sin\theta\cos\theta)/4$<br>$I_y = a^4(\theta + \sin\theta\cos\theta)/4$ | $r_x^2 = \dfrac{a^2}{4}\,\dfrac{(\theta - \sin\theta\cos\theta)}{\theta}$<br>$r_y^2 = \dfrac{a^2}{4}\,\dfrac{(\theta + \sin\theta\cos\theta)}{\theta}$ | $I_{x_c y_c} = 0$<br>$I_{xy} = 0$ |
| <br>Circular Segment | $A = a^2\left(\theta - \dfrac{\sin 2\theta}{2}\right)$<br>$x_c = \dfrac{2a}{3}\,\dfrac{\sin^3\theta}{\theta - \sin\theta\cos\theta}$<br>$y_c = 0$ | $I_x = \dfrac{Aa^2}{4}\left[1 - \dfrac{2\sin^3\theta\cos\theta}{3\theta - 3\sin\theta\cos\theta}\right]$<br>$I_y = \dfrac{Aa^2}{4}\left[1 + \dfrac{2\sin^3\theta\cos\theta}{\theta - \sin\theta\cos\theta}\right]$ | $r_x^2 = \dfrac{a^2}{4}\left[1 - \dfrac{2\sin^3\theta\cos\theta}{3\theta - 3\sin\theta\cos\theta}\right]$<br>$r_y^2 = \dfrac{a^2}{4}\left[1 + \dfrac{2\sin^3\theta\cos\theta}{\theta - \sin\theta\cos\theta}\right]$ | $I_{x_c y_c} = 0$<br>$I_{xy} = 0$ |
| <br>Parabola | $A = 4ab/3$<br>$x_c = 3a/5$<br>$y_c = 0$ | $I_{x_c} = I_x = 4ab^3/15$<br>$I_{y_c} = 16a^3b/175$<br>$I_y = 4a^3b/7$ | $r_{x_c}^2 = r_x^2 = b^2/5$<br>$r_{y_c}^2 = 12a^2/175$<br>$r_y^2 = 3a^2/7$ | $I_{x_c y_c} = 0$<br>$I_{xy} = 0$ |




3. **Given:** A 6' by 4' rectangle is topped by a semi-circular area as shown.

**Find:** Location of the centroid of the entire area by the composite area method.

Note: Centroid of a half-circle is known to lie a distance of $\frac{4R}{3\pi}$ from the center of the circle.




<u>Solution:</u>

$$\bar{x} = \frac{\Sigma A_i x_i}{\Sigma A_i} = \frac{\overset{\text{half circle}}{A_1 x_1} + \overset{\text{rectangle}}{A_2 x_2}}{A_1 + A_2} = \frac{\frac{1}{2}\pi(2)^2 \times 2 + 6 \times 4 \times 2'}{\frac{1}{2}\pi(2)^2 + 6 \times 4}$$

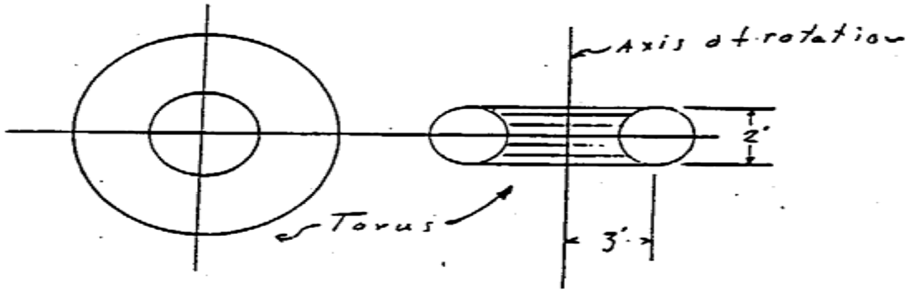
$$= \frac{2[2\pi + 24]}{2\pi + 24} = 2'$$

We could have found $\bar{x} = 2'$ by symmetry (the line $x=2$ is a line of symmetry) or from the fact that the centroid of each part lies on the line $x = 2$.

$$\bar{y} = \frac{\Sigma A_i y_i}{\Sigma A_i} = \frac{A_1 y_1 + A_2 y_2}{A_1 + A_2} = \frac{\frac{1}{2}\pi(2)^2(6 + \frac{4R}{3\pi}) + 24 \times 3}{\frac{1}{2}\pi(2)^2 + 24} = \frac{2\pi(6 + \frac{4 \times 2}{3\pi}) + 72}{2\pi + 24}$$

$$= \frac{2\pi(6.847) + 72}{30.29} = \frac{43 + 72}{30.29} = \frac{115}{30.29} = 3.8'$$

$$\boxed{\begin{array}{l} \bar{x} = 2' \\ \bar{y} = 3.8' \end{array}}$$




4. Given: The torus (doughnut) has dimensions shown:
Find: a) The surface area of the torus.
b) The volume of the torus.

Solution:

a) By the first Pappus theorem,

$A = \theta L \bar{y}$

where $\theta$ = angle of rotation (here $\theta = 2\pi$)
$L$ = length of plane curve
$\bar{y}$ = distance from centroid of line to axis of rotation

$\therefore A = 2\pi(2\pi)(3) = 12\pi^2 = \boxed{115 \text{ sq ft}}$

b) By the second Pappus theorem,

$V = \theta A \bar{y}$

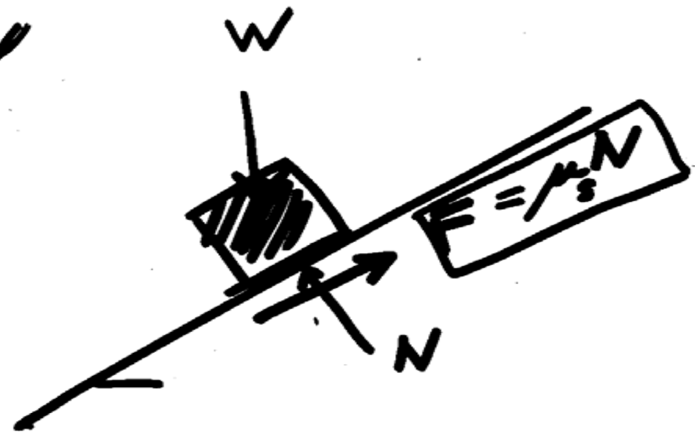
where $\theta$ = same as above (here $\theta = 2\pi$)
$A$ = plane area rotated
$\bar{y}$ = distance from centroid of area to axis of rotation

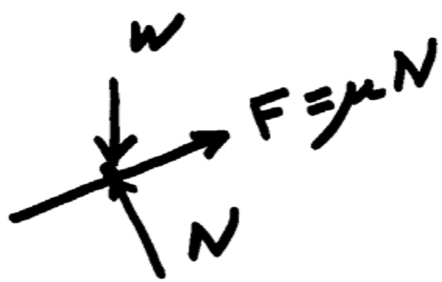
$\therefore V = 2\pi(\pi)(3) = 6\pi^2 = \boxed{59.4 \text{ cu ft}}$

$$\Sigma F_x = 0$$

$$\Sigma F_y = 0$$

$$F_s = \mu_s N$$

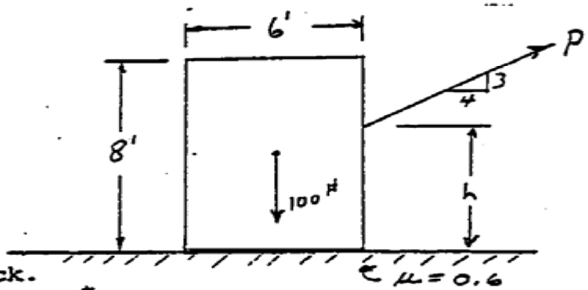


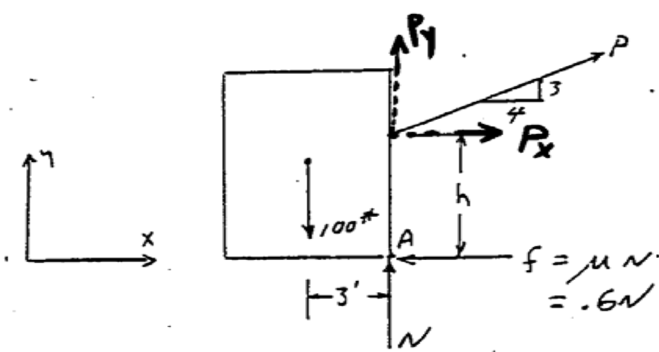

## E. Friction Problems

**Given:** A force P is to be applied to the 6′ x 8′ block as high as possible without tipping the block and just start the block to slide.

**Find:** Force P and height h for a coefficient of friction of $\mu = 0.6$.




**Solution:** Free body is the block.




$$\overset{+}{\rightarrow}\Sigma F_x = 0$$

$$\tfrac{4}{5}P - 0.6N = 0 \quad \ldots \quad (1)$$

$$+\!\uparrow \Sigma F_y = 0$$

$$\tfrac{3}{5}P + N - 100 = 0 \quad \ldots \quad (2)$$

$$\curvearrowleft\!+\ \Sigma M_A = 0$$

$$3 \times 100 - \tfrac{4}{5}P \times h = 0 \quad \ldots \quad (3)$$

Solving (1) & (2) simultaneously, we find

P = 51.7# and N = 69# (notice that N ≠ 100#)

Then from (3), we find

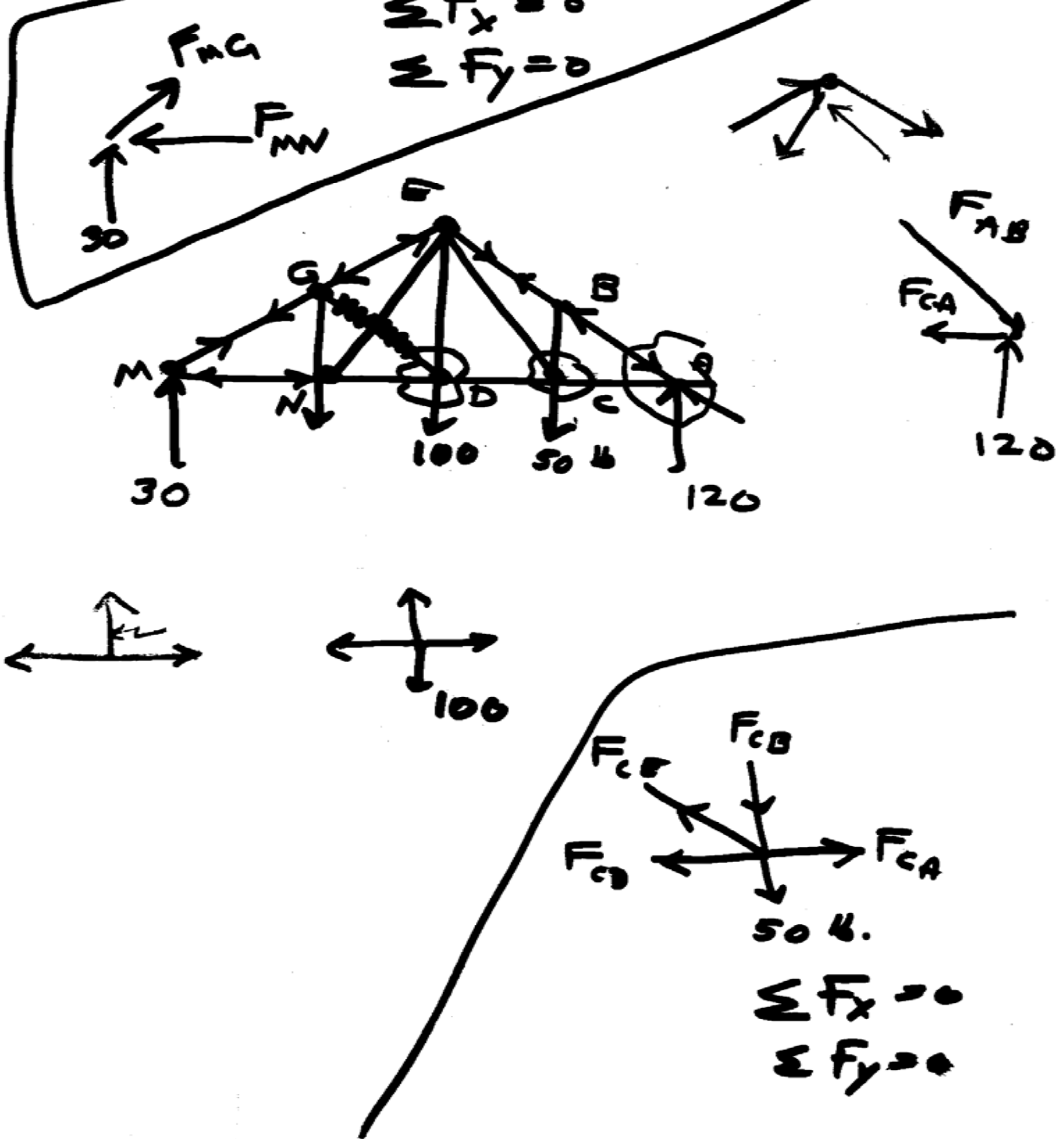
h = 7.25′

$$\boxed{\begin{array}{l} P = 51.7\# \\ h = 7.25' \end{array}}$$

# TRUSSES

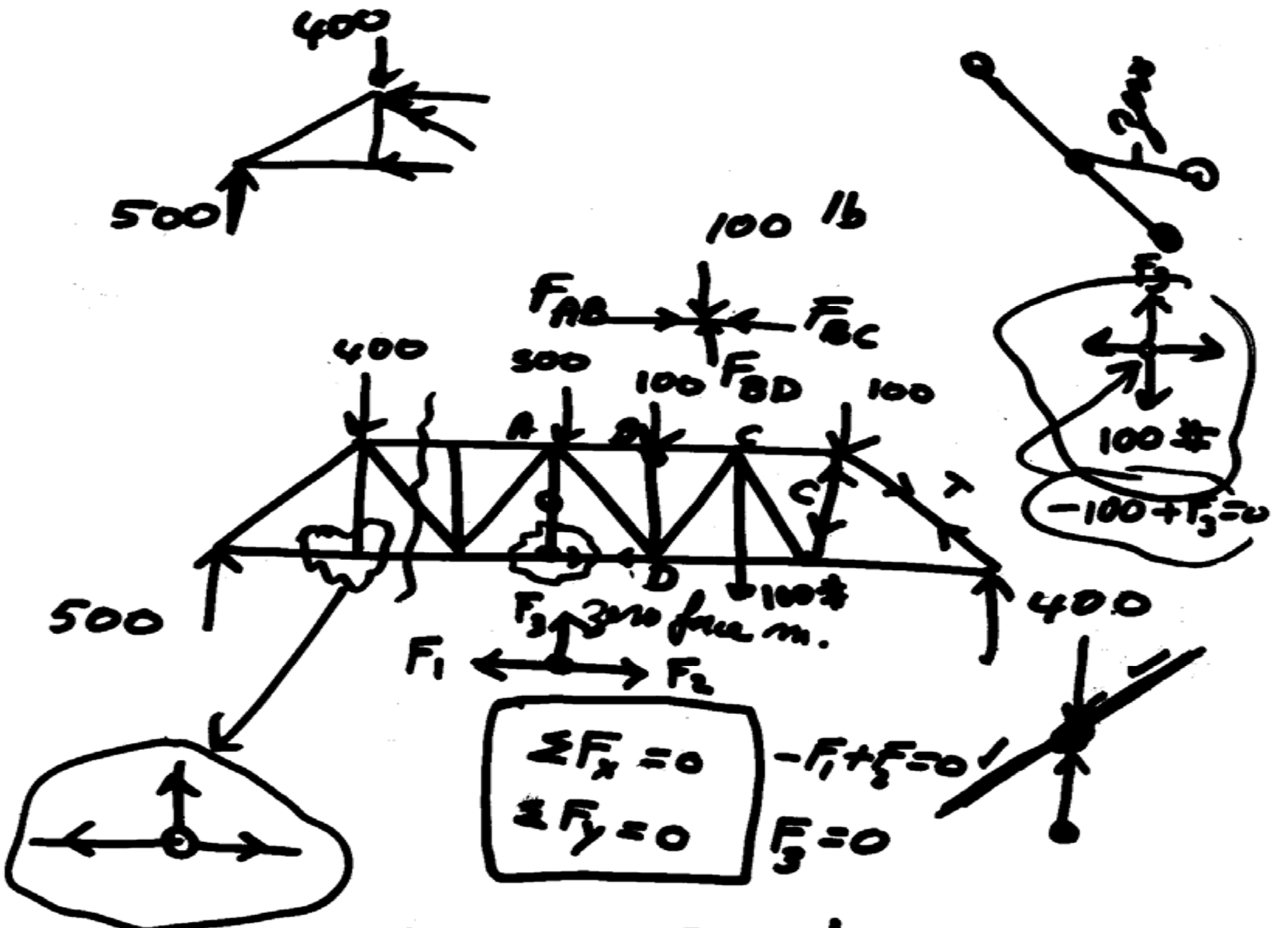
$\Sigma F_x = 0$

$\Sigma F_y = 0$



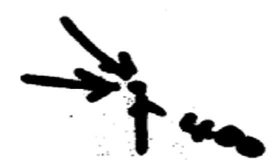

$\Sigma F_x = 0$

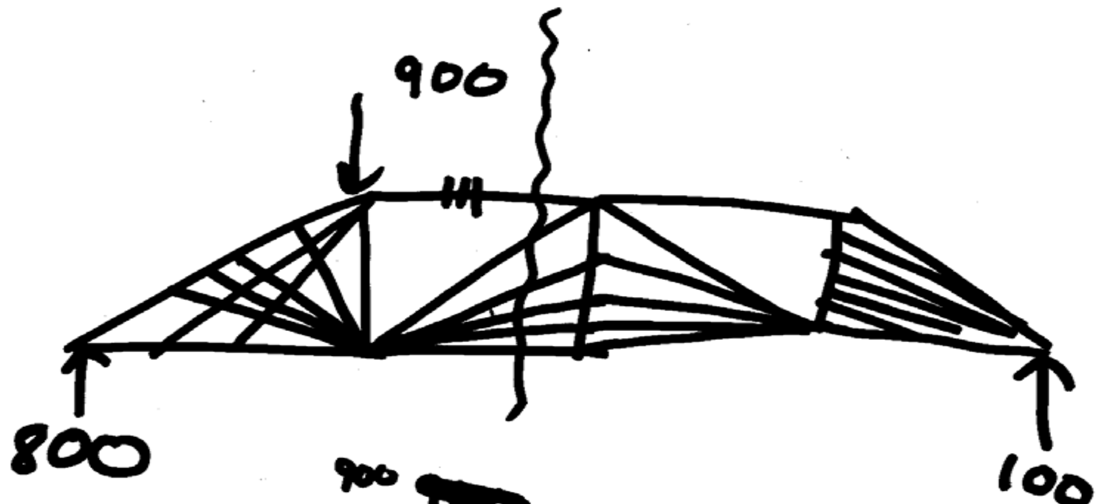
$\Sigma F_y = 0$

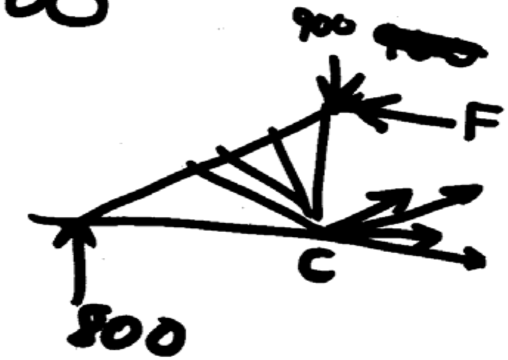



1- Method of Joints

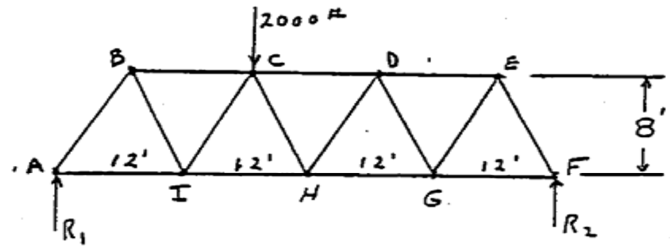
2- Method of Sections




900
800
100


900
F
C
800

## F. Structures




1. Given: The simply supported truss shown is loaded by a 2000# vertical load at point C.

   Find: a) Reactions $R_1$ and $R_2$.
   b) Force in members EF and GF by the methods of joints.
   c) Force in members CD, DH, and GH by the methods of sections.

Solution: a) Free body is entire truss (Picture above)

$$\overset{+}{\curvearrowleft}\Sigma M_A = 0$$

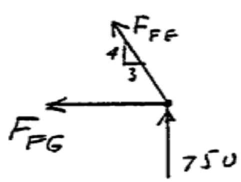
$$48 R_2 - 18 \times 200 = 0 \quad or \quad R_2 = \frac{18 \times 2000}{48} = \boxed{750\# = R_2}$$

$$+\uparrow \Sigma F_v = 0$$

$$R_1 + R_2 - 2000 = 0 \quad or \quad R_1 = 2000 - R_2 = 2000 - 750$$

$$\boxed{R_1 = 1250\#}$$

b) Free body is Joint F.




$$+\uparrow \Sigma F_y = 0$$

$$\frac{4}{5} F_{FE} + 750 = 0 \quad or \quad F_{FE} = -\frac{5}{4} \times 750 = -938\#$$

$$or \quad \boxed{F_{FE} = 938\# \ (C)}$$

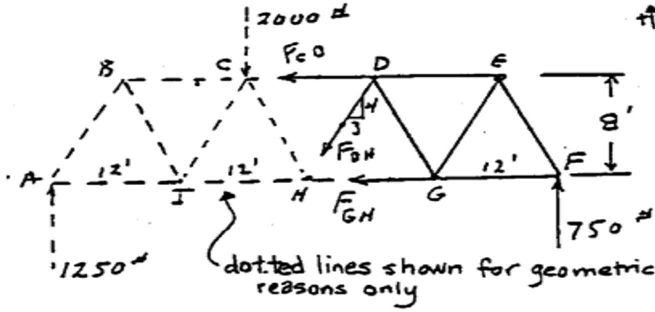
$$\overset{+}{\rightarrow} \Sigma F_H = 0$$

$$-F_{FG} - \frac{3}{5} F_{FE} = 0 \quad or \quad F_{FG} = -\frac{3}{5} F_{FE} = -\frac{3}{5}\left(-\frac{5}{4} \times 750\right)$$

$$= \frac{3}{4} \times 750 = 563\#$$

$$or \quad \boxed{F_{FG} = 563\# \ (T)}$$

c) Free body is right "half" (or section) of the beam (solid lines).




$$\stackrel{+\uparrow}{\Sigma} F_V = 0$$

$$750 - \frac{4}{5} F_{DH} = 0$$

or

$$F_{DH} = \frac{5}{4} \times 750$$

$$= 938\#$$

or $\boxed{F_{DH} = 938\#\ (T)}$

$$\stackrel{\curvearrowleft +}{\Sigma} M_D = 0$$

$$18 \times 750 - F_{GH} \times 8 = 0 \quad \text{or} \quad F_{GH} = \frac{18 \times 750}{8} = 1685\#$$

or $\boxed{F_{GH} = 1685\#\ (T)}$

$$\stackrel{+\rightarrow}{\Sigma} F_H = 0$$

$$-F_{CD} - F_{GH} - \frac{3}{5} F_{DH} = 0 \quad \text{or} \quad F_{CD} = -F_{GH} - \frac{3}{5}\left(\frac{5}{4} \times 750\right)$$

$$= -1685 - 563$$

$$= -2248\#$$

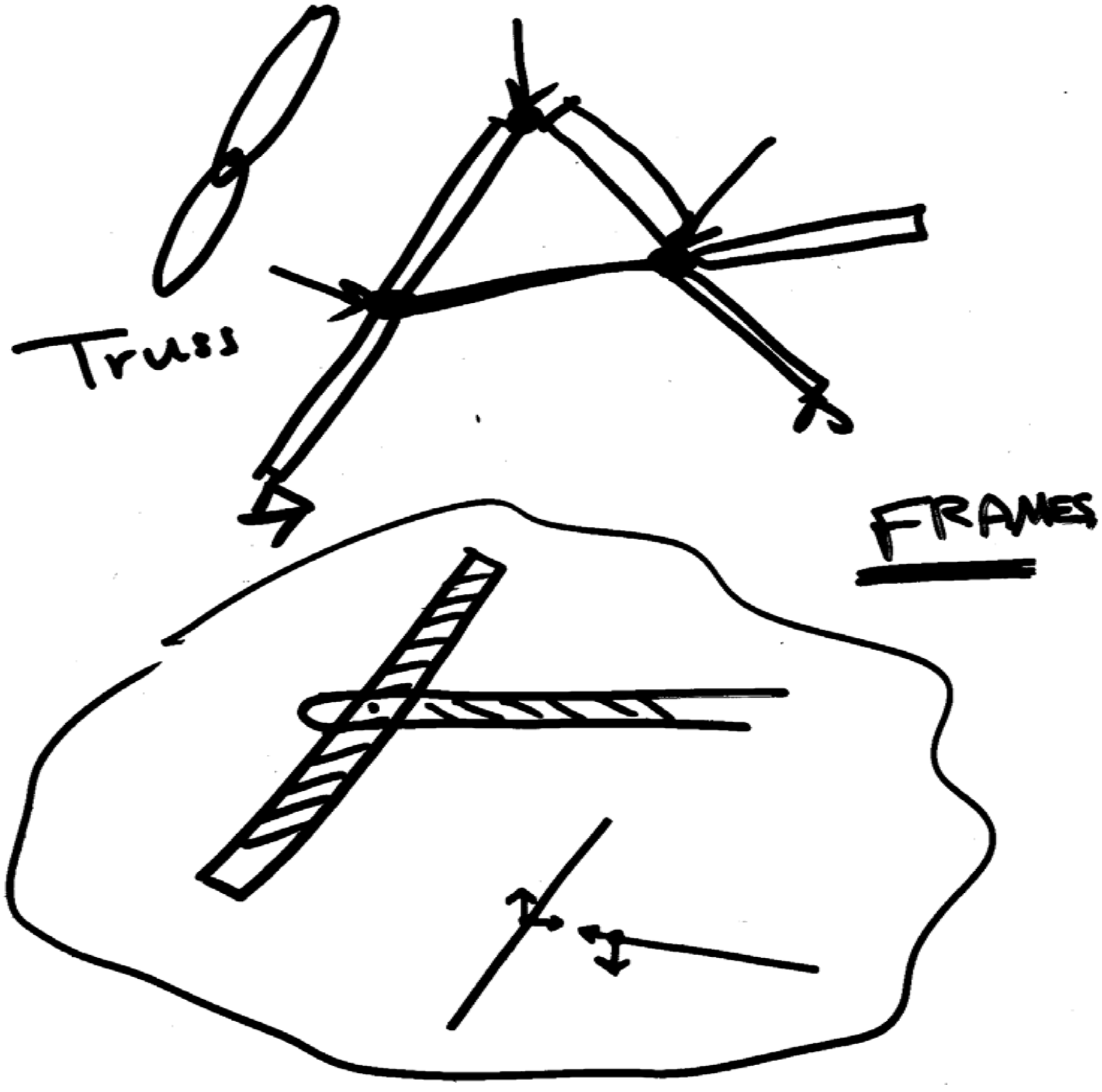
or $\boxed{F_{CD} = 2248\#\ (C)}$

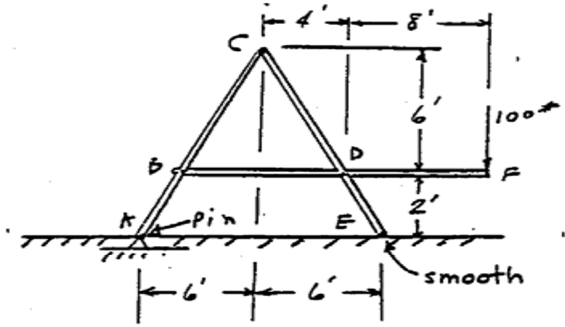
**Note:** We can also find $F_{CD}$ by summing moments with respect to Point H; moment axes do not need to be attached to the free body.

$$\stackrel{\curvearrowleft +}{\Sigma} M_H = 0$$

$$8 F_{CD} + 24 \times 750 = 0 \quad \text{or} \quad F_{CD} = \frac{-24 \times 750}{8} = -2250\#\ (C)$$

Checks solution above.


Truss
FRAMES




2. **Given:** The given frame is made up of continuous members pinned at B, C, and D. The frame is loaded by a 100# force at F and is supported by a pin at A and by the smooth horizontal plane at E.

**Find:** a) Reactions at A and E.
b) Pin forces at B, C, and D.

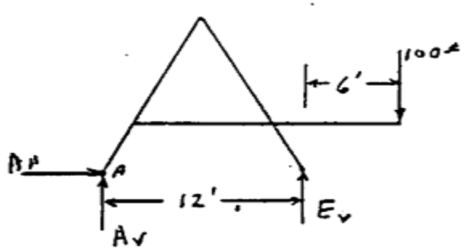
<u>Solution:</u>

a) Free body is the entire frame.

$$\overset{\curvearrowleft +}{\sum} M_A = 0$$

$$12\,E_V - 100 \times 18 = 0$$

$$\text{or } E_V = \frac{18 \times 100}{12} = 150\#$$




$$\overset{+}{\rightarrow}\sum F_H = 0$$

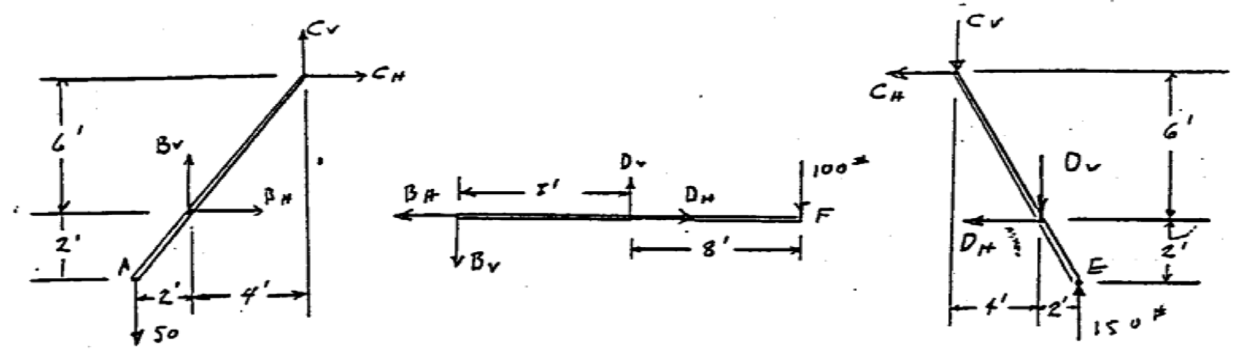
$$A_H = 0$$

$$+\uparrow \sum F_V = 0$$

$$A_V + E_V - 100 = 0$$

$$\text{or } A_V = 100 - E_V = 100 - 150 = -50.$$

$$\therefore \boxed{\begin{array}{l} A_H = 0 \\ E_V = 150\# \uparrow \\ A_V = 50\# \downarrow \end{array}}$$




**Free body is member BDF.**

$$\overset{+}{\curvearrowleft}\sum M_B = 0 \Rightarrow 8D_v - 16\times 100 = 0 \quad \text{or} \quad \boxed{D_v = 200\#}$$

$$+\!\uparrow\sum F_v = 0 \Rightarrow D_v - B_v - 100 = 0 \quad \text{or} \quad B_v = D_v - 100 = 200 - 100 = 100\#$$

$$\boxed{B_v = 100\#}$$

**Free body is member ABC.**

$$+\!\uparrow\sum F_v = 0$$

$$B_v + C_v - 50 = 0 \quad \text{or} \quad C_v = 50 - B_v = 50 - 100 = -50$$

$$\boxed{C_v = 50\downarrow \text{ on member ABC}}$$

$$\overset{+}{\curvearrowleft}\sum M_c = 0$$

$$50\times 6 + 6B_H - 4B_v = 0$$

$$B_H = \frac{4B_v - 300}{6} = \frac{4(100) - 300}{6} = \frac{100}{6} = 16.65\#$$

$$\boxed{B_H = 16.65\#}$$

$$\overset{+}{\rightarrow}\sum F_H = 0$$

$$C_H + B_H = 0 \quad \text{or} \quad C_H = -B_H = -16.65\#$$

$$\boxed{C_H = \overleftarrow{16.65}\# \text{ on member ABC}}$$

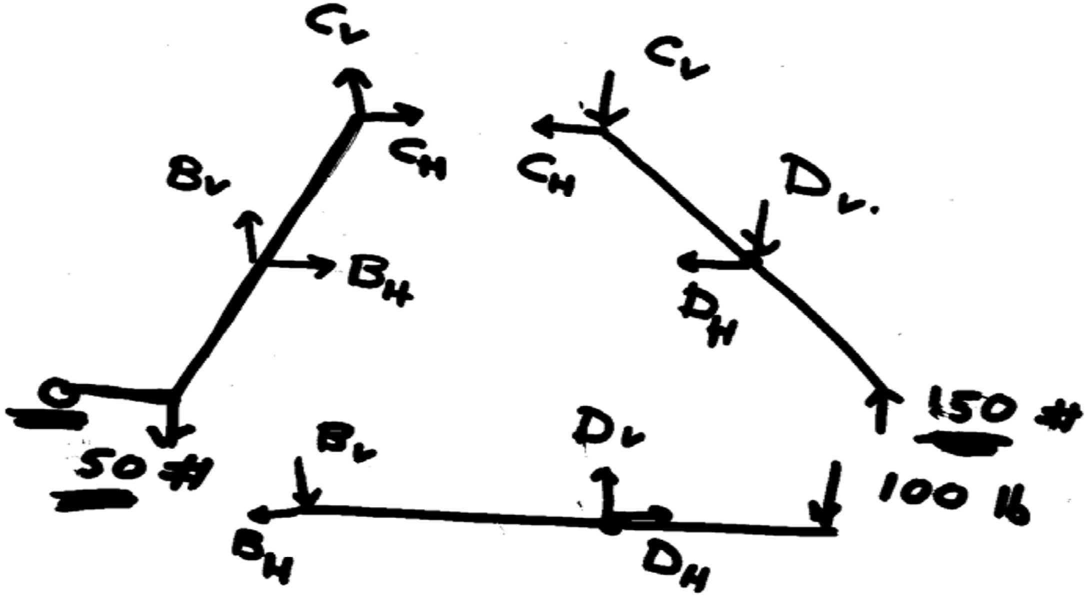
**Free body is member BDF again.**

$$\overset{+}{\rightarrow}\sum F_H = 0$$

$$D_H - B_H = 0 \quad \text{or} \quad D_H = B_H = 16.65$$

$$\text{or} \quad \boxed{D_H = \overrightarrow{16.65}\# \text{ on member BDF}}$$

**Note:** Checks can be made now, using member CDE as a free body.


Cv
CH
Bv
BH
50 #
Cv
CH
Dv.
DH
150 #
Bv
Dv
100 lb
BH
DH